THE OPEN UNIVERSITY
A SCIENCE FOUNDATION COURSE

UNIT 30 QUANTUM MECHANICS: AN INTRODUCTION

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1.5

STUDY GUIDE

This Unit consists of three components—the text, an AV sequence and a TV programme. There are fewer pages than in most other Units, and you should find that your study time is somewhat shorter. However, this Unit contains several subtle concepts that normally each take some time to be absorbed. We should also like to point out that the Unit contains some historical background to the development of quantum physics—you do not have to remember *any* of this historical material.

You should work through the AV sequence 'Wavefunctions of matter' (Tape 5, Side 1, Band 1), which concerns quantum mechanical waves, when you reach Section 3.2 of the text. The sequence should take you about 40 minutes to study.

The TV programme, 'Quantum physics—electrons and photons', can be watched with profit at any stage in your studies of the Unit. Notes for the programme are included in Section 2.

1 INTRODUCTION TO THE MODERN PHYSICS BLOCK (UNITS 30–32)

Common sense is something that almost everyone thinks they have in abundance. Although it is a difficult (or perhaps impossible) concept to define, common sense is something that we are often urged to use and to trust.

We have frequently appealed to *your* common sense when we have presented scientific arguments, for example by prefacing apparently uncontentious statements with 'obviously' or 'clearly'. You may have found this irritating, but you would surely agree that *these* two statements are obviously true:

1 When someone walks through a doorway without touching the door frame (Figure 1a), the width of the doorway does not affect the walker's subsequent path.

2 When a stone is dropped down a well (Figure 1b), it is possible to predict for certain where the stone will strike the bottom of the well.

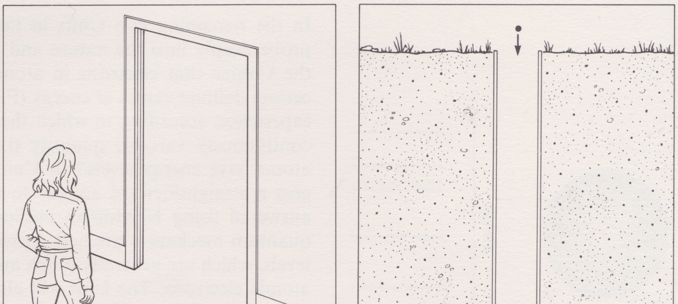

FIGURE 1 Two situations that appear to be amenable to commonsense analyses: (a) someone walking through a doorway; (b) a stone falling down a well.

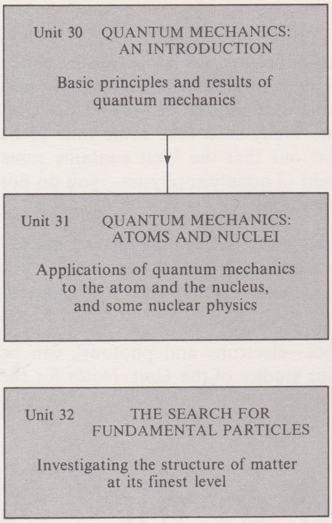
Unit 30 QUANTUM MECHANICS: AN INTRODUCTION
Basic principles and results of quantum mechanics
Unit 31 QUANTUM MECHANICS: ATOMS AND NUCLEI
Applications of quantum mechanics to the atom and the nucleus, and some nuclear physics
Unit 32 THE SEARCH FOR FUNDAMENTAL PARTICLES
Investigating the structure of matter at its finest level

FIGURE 2 The modern physics Block of the Course.

These are uncontentious statements, are they not? Maybe—but they are both believed to be wrong. The theory that implies that the statements are not correct is quantum mechanics, a theory that revolutionized our understanding of the behaviour of matter when it was formulated in the mid-1920s. Until then, motion was believed to be described adequately by the theory of mechanics proposed by Newton almost 250 years before, in 1687 (Unit 3). Newton's laws had given an extremely successful description of matter—the laws could be applied equally well to objects in the everyday world (such as an apple falling from a tree) and to celestial objects (such as the Moon orbiting the Earth). However, soon after the beginning of the 20th century, it was found experimentally that these laws could not account satisfactorily for the behaviour of small-scale matter, such as atoms and subatomic particles. Quantum mechanics does allow this behaviour to be understood, and it can explain how large-scale matter behaves, so this theory has superseded Newtonian mechanics as the best available theory of matter.

This Unit, the first in the modern physics Block (Figure 2), is concerned with some of the most basic concepts and principles of quantum mechanics. It begins by revising the models that are needed to understand the behaviour of electromagnetic radiation (e.g. light), and then goes on to consider the behaviour of matter (electrons, protons, neutrons, etc.). You will see that, contrary to expectations based on everyday experience, each sample of matter has an associated wave. The quantum mechanical interpretation of this wave causes us to abandon the simple commonsense picture of electrons, protons and neutrons, etc., as particles with perfectly defined positions and velocities. According to quantum mechanics, a much more subtle visualization of such microscopic matter is required.

We then move on to discuss one of the most crucial principles of quantum mechanics—the Heisenberg uncertainty principle. This principle radically changed the physicist's view of the concept of measurement. For example, according to Heisenberg, the accuracy with which the *position* of an electron is known unavoidably restricts the accuracy with which its *momentum* can be known simultaneously (and vice versa). This is incomprehensible in terms of Newtonian ideas.

Finally, some of the basic concepts of quantum mechanics are applied to matter in the everyday world in order to see how certain predictions of the theory differ from those based on common sense. When you have completed the Unit, you will have seen that the behaviour of matter is more complicated than it appears to be in everyday life, and you will be able to understand why the two statements made at the beginning of this Introduction are incorrect.

In the remaining two Units in the Block, quantum ideas will be used to probe deeply into the nature and behaviour of matter. You saw earlier in the Course that electrons in atoms have energy levels, that is, they have certain definite values of energy (Figure 3). This is in contrast with everyday experience, according to which the energy of an object is perceived to be a continuously variable quantity (i.e. *not* quantized). Why do electrons in atoms have energy levels? In Unit 31, quantum mechanics will be used to give a straightforward answer to this question, which cannot possibly be answered using Newtonian mechanics. You will also see in Unit 31 that quantum mechanics predicts correctly that atomic *nuclei* also have energy levels, which are generally much more widely spaced in energy than those of atomic electrons. The Unit will also be concerned with some other aspects of the behaviour and properties of nuclei. In particular, radioactive decays will be discussed and we shall explain briefly, in biological terms, why the products of radioactive processes can be hazardous. Also, nuclear fusion and fission will be considered and this will lead us to a brief discussion of nuclear energy.

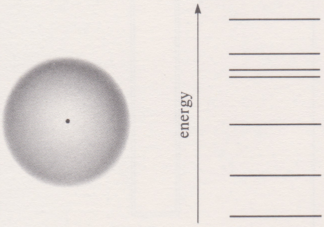

FIGURE 3 Electrons in atoms have energy levels, i.e. discrete values of energy.

In Unit 32 we shall discuss particle physics, the branch of physics that concerns the behaviour and interactions of subatomic particles, such as electrons, protons and neutrons. You will see that the structure of matter

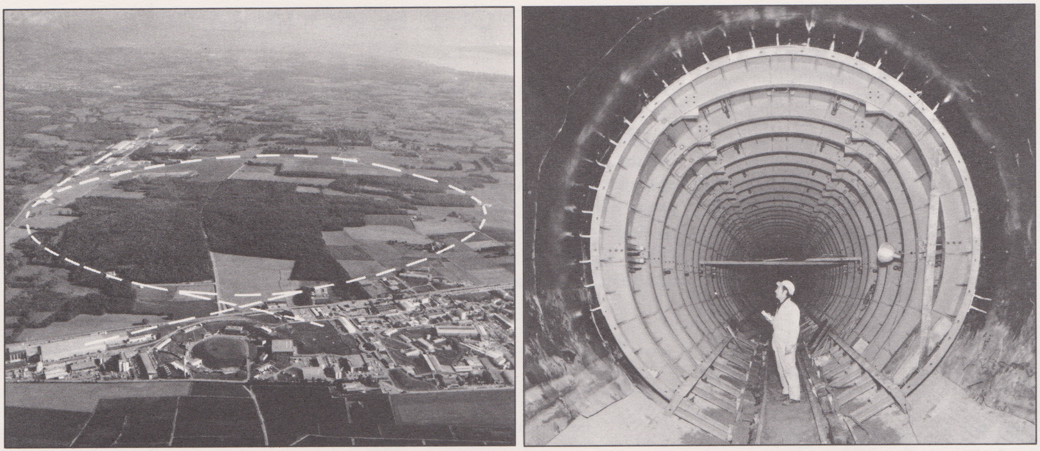
(a) (b)

FIGURE 4 (a) The outline of the underground super proton synchroton (SPS) at Geneva. (b) The tunnel of the SPS when under construction. (Photos courtesy CERN/Science Photo Library)

can be probed at extremely small distances (typically less than 10^{-15} m, a thousand-million-millionth of a metre) using accelerators that produce beams of particles travelling at speeds extremely close to that of light in a vacuum (Figure 4). In order to understand the behaviour of matter that travels at these speeds, it is essential to use the special theory of relativity. The details of this theory will not be considered in this Block, but in Units 31 and 32 some simple results that follow from the theory will be used. In discussions of matter in Unit 30, only matter that is travelling at speeds much less than that of light in a vacuum will be considered, so relativistic complications will not arise.

The results of experiments done with particle accelerators have shown that the most fundamental description of the structure of matter should be given not in terms of atoms and molecules, but in terms of types of fundamental particle known as leptons, quarks and gauge bosons. As you will see in Unit 32, the behaviour of some of these particles transcends everyday experience: some fundamental particles have unusual properties such as ‘strangeness’ and ‘charm’; some can transform spontaneously into other particles; some are detected only indirectly and are expected *never* to be directly observed!

As you can see, we shall be covering some extraordinary material in this Block. When you have finished the three Units, you may well be quite reluctant to use your common sense when you are analysing scientific problems. You may even agree with Albert Einstein, who once said that ‘common sense is the deposit of prejudice laid down in the mind before the age of eighteen’.

2 DESCRIBING THE BEHAVIOUR OF ELECTROMAGNETIC RADIATION AND MATTER (TV PROGRAMME)

In this Section, we shall consider the behaviour of electromagnetic radiation and of matter. First, the two models of electromagnetic radiation that you met in Unit 10 are revised, then models of matter (electrons, protons, neutrons, etc.) are examined. The material in this Section constitutes the broadcast notes for the TV programme ‘Quantum physics—electrons and photons’.

PHOTON

PHOTOELECTRIC EFFECT

2.1 MODELS OF ELECTROMAGNETIC RADIATION

As you saw in Unit 10, light that can be seen by human beings is a particular type of electromagnetic radiation ('radiation' for short) that has wavelengths between approximately 400 nm and approximately 700 nm (remember that 1 nm = 10^{-9} m). There are other types of radiation (for example, γ-rays, X-rays, and microwaves) and each one has a characteristic range of wavelengths (Figure 5). Two entirely different models are required to understand the behaviour of electromagnetic radiation: a wave model, which describes the radiation's propagation, and a particle model, which describes its interactions.

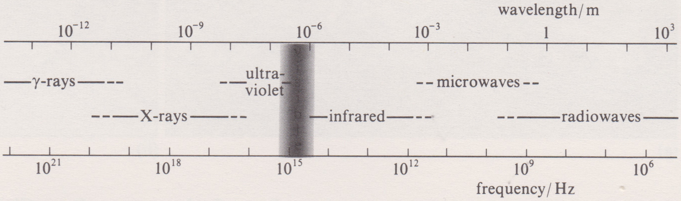

FIGURE 5 The electromagnetic spectrum (note that the boundaries between different regions are not well defined).

2.1.1 WAVE MODEL OF ELECTROMAGNETIC RADIATION

The wave model of radiation is used almost continuously in the analysis of the physics experiment at Summer School. You may remember that in the experiment, a collimated beam of light from a discharge tube propagates towards a diffraction grating, and the diffracted light is observed using an adjustable telescope (Figure 6).

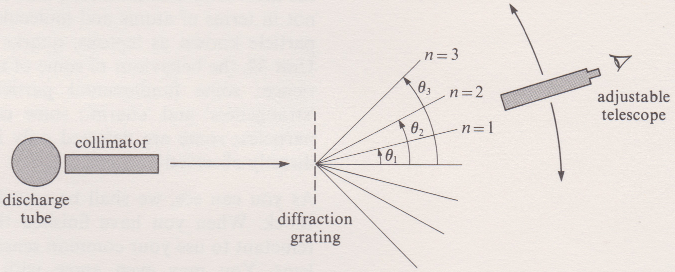

FIGURE 6 When light of wavelength λ impinges on a grating with spacing d, the light is diffracted and the maxima occur at angles θ_n, where $n\lambda = d \sin \theta_n$. This apparatus is used in the physics experiment at Summer School.

For radiation of wavelength λ propagating towards a diffraction grating of spacing d, the angle θ_n of the nth diffracted beam is given by

$$n\lambda = d \sin \theta_n \tag{1}$$

an expression that was derived in Unit 10 using a wave model. For the $n = 1$ beam, Equation 1 says that

$$\lambda = d \sin \theta_1, \quad \text{i.e. } \sin \theta_1 = \frac{\lambda}{d}$$

and for the $n = 2$ beam, Equation 1 says that

$$2\lambda = d \sin \theta_2, \quad \text{i.e. } \sin \theta_2 = 2\left(\frac{\lambda}{d}\right)$$

It is important to remember that diffraction effects are most easily observed (i.e. θ_1 is most easily measurable) if the grating spacing d is not much larger than the wavelength λ of the radiation (Figure 7). If the wavelength is much less than the grating spacing, the diffraction angle θ_1 will be negligibly small—the radiation will appear to propagate straight through (Figure 8).

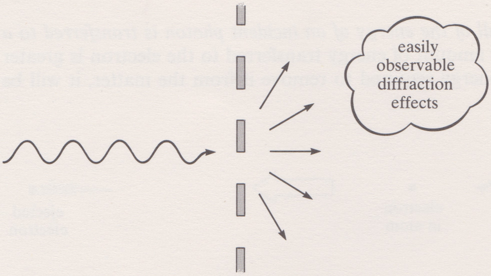

FIGURE 7 Diffraction effects are easily observable if the wavelength of the incident radiation is not much less than the grating spacing.

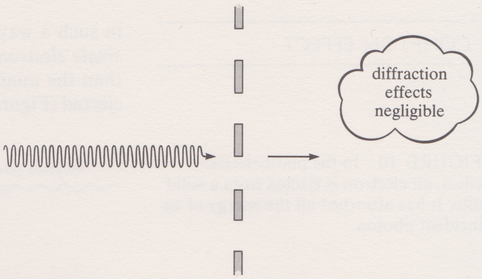

FIGURE 8 If the wavelength is much less than the spacing, diffraction effects will be negligible.

ITQ 1 A beam of red light with wavelength 700 nm (7.00×10^{-7} m) impinges on a diffraction grating with a spacing of 1 400 nm (1.40×10^{-6} m). Then, in a separate experiment, a beam with the same wavelength is made to impinge on another grating, with a spacing of 1 mm (1.00×10^{-3} m).

(a) In which of the two experiments will diffraction be most clearly evident? (Try to answer this without doing any calculations.)

(b) Calculate the value of θ_1, the angle of the $n = 1$ diffraction maximum, for each experiment. Check that your answer to this part of the question is consistent with your answer to part (a).

You may remember from Unit 10 that, according to the wave model of electromagnetic radiation, a beam of this radiation is associated with an electromagnetic wave. The frequency f of the electromagnetic wave (in a vacuum) is related to its wavelength λ by the relation

$$f = c/\lambda \tag{2}$$

where c is the speed of light (and of any other type of electromagnetic radiation) in a vacuum, approximately $3.00 \times 10^8 \text{ m s}^{-1}$.

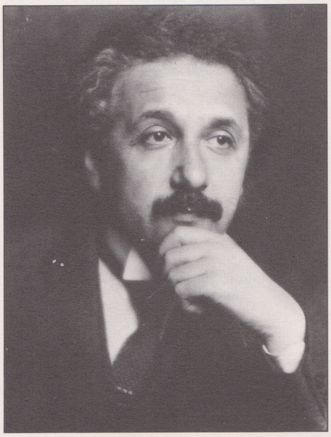

FIGURE 9 Albert Einstein (1879–1955) is widely regarded as the most outstanding scientist of the twentieth century. In January 1919, when he and his first wife divorced, he promised that he would give her the money he would receive when his Nobel Prize was awarded. He was as good as his word: four years later, after he won the 1921 Nobel Prize for physics, he gave her the entire 121 572 Kronor (about $32 000). He was awarded the Prize for his 'services to theoretical physics, and especially for his discovery of the law of the photoelectric effect'.

2.1.2 PARTICLE MODEL OF ELECTROMAGNETIC RADIATION

When electromagnetic radiation interacts (i.e. exchanges energy) with matter, its behaviour can be described adequately only in terms of a particle model. According to this model, a beam of electromagnetic radiation is considered as a stream of quanta that each have energy and momentum. Each quantum is called a **photon**.

For a beam of radiation with frequency f, each photon has the same energy E given by the Planck–Einstein formula

$$E = hf \tag{3}$$

where h is Planck's constant, the value of which is approximately 6.63×10^{-34} J s. Note that it is easy to state Equation 3 in an alternative form using the expression for the frequency f given in Equation 2:

$$E = \frac{hc}{\lambda} \tag{4}$$

ITQ 2 Consider again the beam of red light of wavelength 700 nm (7.00×10^{-7} m) described in ITQ 1. What is the energy of each photon in this beam of radiation? (Calculate the answer to two significant figures.)

In Unit 10, you met two types of interaction between radiation and matter—the photoelectric effect and the Compton effect. In the **photoelectric effect**, first explained by Einstein (Figure 9), radiation interacts with matter in such a way that *all of the energy of an incident photon is transferred to a single electron.* If the amount of energy transferred to the electron is greater than the minimum energy required to remove it from the matter, it will be ejected (Figure 10).

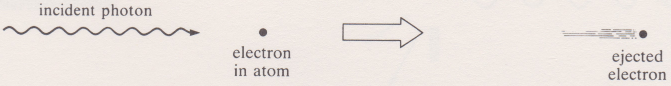
incident photon
electron in atom
ejected electron

FIGURE 10 In the photoelectric effect, an electron is ejected from a solid after it has absorbed all the energy of an incident photon.

COMPTON EFFECT

In the **Compton effect**, radiation is *scattered* by an electron—in this case, *not all of the photon's energy is transferred to the electron.* The effect is illustrated in Figure 11: the incoming photon collides with an electron, which thereby acquires kinetic energy. The photon's energy is therefore reduced.

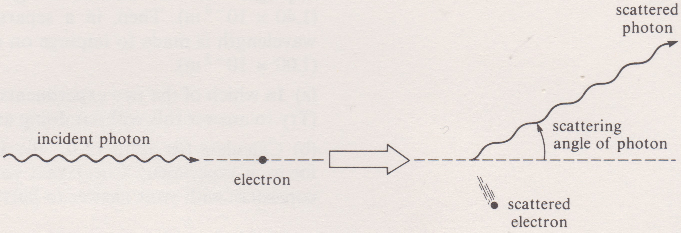
scattered photon
scattering angle of photon
incident photon
electron
scattered electron

FIGURE 11 In the Compton effect, an electron is scattered by a photon.

☐ Is the wavelength of the scattered radiation shorter than, longer than or the same as that of the incident radiation?

■ Longer. Because the energy of the scattered photon is *lower* than that of the incident photon, the wavelength of the scattered radiation must be *higher* than that of the incident radiation. (Remember from Equation 4 that the energy E of a photon is inversely proportional to the wavelength λ of the radiation.)

In the TV programme, we show how the Compton effect can be studied experimentally, using the apparatus illustrated schematically in Figure 12.

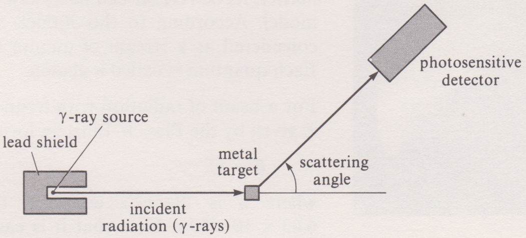
photosensitive detector
γ-ray source
lead shield
metal target
scattering angle
incident radiation (γ-rays)

FIGURE 12 A diagram of the apparatus used to demonstrate the Compton effect in the TV programme 'Quantum physics—electrons and photons'.

A collimated beam of γ-rays propagates towards a metal target, and the scattered radiation is detected using a photosensitive detector. The photons are scattered at all angles between zero degrees (the 'straight-through' position, which corresponds to no scattering) and 180°, which is the scattering angle of photons that are reflected directly back in their tracks. At each angle, the scattered photons are found to have a characteristic energy and, moreover, the larger the scattering angle, the lower the energy of the scattered photons that are detected at that angle (Figure 13).

The data can be understood only by assuming that each photon has momentum. As you saw in Unit 10, the momentum of a photon has a direction that is the same as that in which the radiation propagates, and the

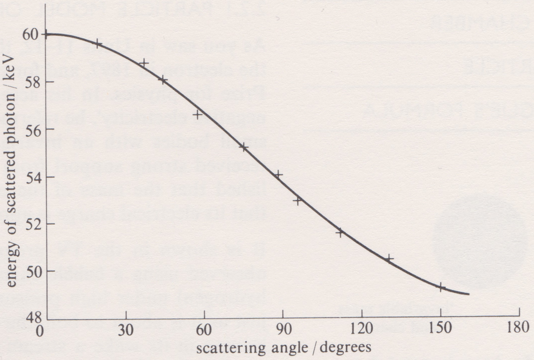

FIGURE 13 In the Compton effect, the greater the scattering angle of the incident photon (Figure 12), the more energy it loses. The data in this Figure refer to the experiment demonstrated in the TV programme 'Quantum physics—electrons and photons'.

magnitude p of the photon's momentum is given by

$$p = E/c \tag{5}$$

where E is its energy (remember $E = hf$, Equation 3) and where c is the speed of light in a vacuum. If Equation 5 is used in conjunction with the laws of conservation of energy and momentum, the dependence of a photon's energy loss on its scattering angle can be derived theoretically (we shall not show here how this is done). The theory accounts beautifully for all data on the effect (e.g. Figure 13).

FIGURE 14 Arthur Compton (1892–1962) was awarded a Nobel Prize for physics in 1927 for 'his discovery of the effect named after him'. He did most of this work at the physics department of Washington University in St Louis, Missouri.

As you saw in Unit 10, the validity of the particle model of light was established by experiments on the Compton effect. These experiments were done by Arthur Compton (Figure 14), who had also been one of the first to formulate a theory of the effect.

SAQ 1 In Table 1, the photoelectric effect is compared with the Compton effect. Fill in the four gaps in the Table.

TABLE 1 Comparison between the photoelectric effect and the Compton effect (SAQ 1)

	Photoelectric effect	Compton effect
type of process: absorption or scattering?		
proportion of the incident photon's energy transferred to the electron		not all of the photon's energy is transferred
law(s) required to understand the process	conservation of energy	

2.2 MODELS OF MATTER

You have just seen that the behaviour of electromagnetic radiation can be described in terms of two models—a wave model, which describes the radiation's propagation, and a particle model, which describes its interactions. But what about the behaviour of *matter*, for example the electron? Surely, a particle model is all that is needed to describe its behaviour?

Remember that in this Unit, we shall consider only matter that is travelling at speeds much less than the speed of light in a vacuum. In this way, the complications of the theory of relativity are avoided.

BUBBLE CHAMBER

FREE PARTICLE

DE BROGLIE'S FORMULA

2.2.1 PARTICLE MODEL OF MATTER

As you saw in Units 11–12, the English physicist J. J. Thomson discovered the electron in 1897, and for this discovery he was awarded the 1906 Nobel Prize for physics. In his acceptance speech, which he entitled 'Carriers of negative electricity', he referred to electrons as 'particles' and as 'corpuscles, small bodies with an invariable mass and charge' (Figure 15). His ideas received strong support from experiments and by 1916 it had been established that the mass of the electron is approximately 9.1×10^{-31} kg and that its electrical charge is approximately -1.6×10^{-19} C.

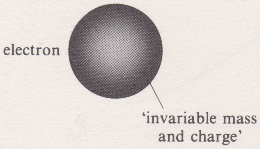

FIGURE 15 According to J. J. Thomson, an electron is a 'small corpuscle with invariable mass and charge'.

It is shown in the TV programme that the path of an electron can be observed using a **bubble chamber**. This device contains a liquid (normally hydrogen) under high pressure. If an electron is propelled into the liquid just as it is about to boil, the electron can ionize the atoms it collides with, leaving in its wake a stream of tiny bubbles that can be illuminated and photographed to show the path of the electron (Figure 16). The leaving of a clearly defined track is, of course, just the behaviour you should expect if the electron were a particle, as Thomson envisaged.

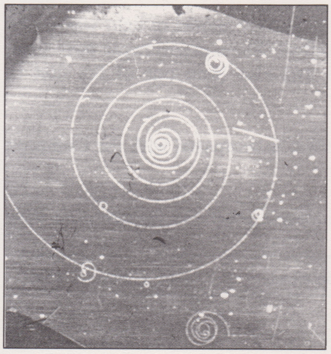

FIGURE 16 The track left by an electron in the liquid hydrogen in a bubble chamber. The track is curved because there is a magnetic field perpendicular to the electron's direction of motion.

The evidence that supports the idea that the electron is a particle is derived from experiments on its *interactions*: for example, its tracks in liquid hydrogen are observable because it interacts with hydrogen atoms. Experiments on other types of matter (e.g. the proton) confirm that their interactions can be understood only if a particle model is used.

So much for the interactions of matter. Let's now move on to discuss the propagation of matter—the motion of matter when it *does not* exchange energy with other matter.

2.2.2 DE BROGLIE'S WAVE MODEL OF MATTER

In September 1923, Prince Louis de Broglie, a young graduate student at the Sorbonne, came up with a truly extraordinary idea. He suggested that the phenomenon of the wave–particle duality might apply not only to radiation—it might also be exhibited by matter. Perhaps, de Broglie proposed, although matter behaves as a particle when it interacts (i.e. exchanges energy with other matter or radiation), it might behave as a *wave* when it propagates (i.e. when it moves without exchanging energy).

Just over a year later, he submitted his doctoral thesis 'Recherches sur la Théorie des Quanta', which contained the details of his theory. The key idea that he presented was that *every sample of matter has an associated wave*. (This wave is *not* an electromagnetic wave—we shall discuss its nature in the AV sequence.) For a particle that is not subject to a net force, that is a **free particle**, he predicted that the wavelength λ_{dB} of the associated wave is given by Planck's constant h divided by the magnitude p of the particle's momentum:

$$\boxed{\lambda_{\mathrm{dB}} = \frac{h}{p}} \tag{6}$$

This is known as **de Broglie's formula**. It was de Broglie's innovation to apply this relationship to matter, but the expression also applies to radiation, as you will see in ITQ 3.

ITQ 3 Using Equations 2 ($c = f\lambda$), 3 ($E = hf$) and 5 ($p = E/c$), show that for a photon with momentum of magnitude p the quantity h/p is equal to the wavelength λ of the radiation.

For matter (as opposed to radiation), the magnitude p of its momentum is given by the product of its mass m and its speed v

$$p = mv \tag{7}$$

as you saw in Unit 3. Substitution of Equation 7 into de Broglie's formula shows that

$$\lambda_{\mathrm{dB}} = \frac{h}{mv} \tag{8}$$

Hence, according to de Broglie, for a free mass *m*, the faster it is travelling (i.e. the higher its speed *v*), the shorter is its associated wavelength (i.e. the lower is its value of λ_{dB}).

The committee that examined de Broglie's thesis praised the originality of his work and duly awarded him a doctorate, but it appears that the examiners did not believe in the physical reality of the newly proposed waves. Other physicists also received the idea with considerable scepticism and, in some cases, derision. In the absence of data to support what was, after all, only a theoretical conjecture, the idea could be ridiculed with impunity. Louis de Broglie was regarded by many as an eccentric and by some as a crank, but he was taken seriously by Einstein, who stressed the importance of doing experiments to investigate the validity of de Broglie's idea.

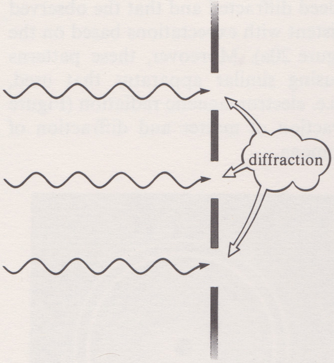

FIGURE 17 Diffraction of matter should be observable if the wavelength of the matter is not much less than the spacing of the grating. (Compare this with Figures 7 and 8.)

How could the idea be checked experimentally? In order to answer this question, think again about the propagation of electromagnetic radiation.

☐ Which type of experiment involving radiation shows that its propagation can be understood only by using a *wave* model?

■ Diffraction experiments. In these experiments, radiation propagates towards a grating whose spacing is not much greater than the wavelength of the radiation (Figure 17).

If each sample of free matter does have an associated wave, then such matter should be noticeably diffracted by a grating with a spacing that is of roughly the same order of magnitude as the matter's wavelength (de Broglie actually pointed this out to the examiners of his thesis!). Consider, for example, a free electron that has been accelerated by a few hundred volts; such an electron has a de Broglie wavelength of approximately 10^{-10} m. Gratings that should noticeably diffract this electron are available in the form of pure metallic crystals which, as you saw in Units 13–14, can be visualized as a regular array of ions in a pool of electrons (Figure 18). Such a structure is closely analogous to that of a conventional optical grating, whose opaque and transparent parts correspond respectively to the ions and the electrons. The average spacing between the ions is, loosely speaking, the 'grating spacing' of the crystal.

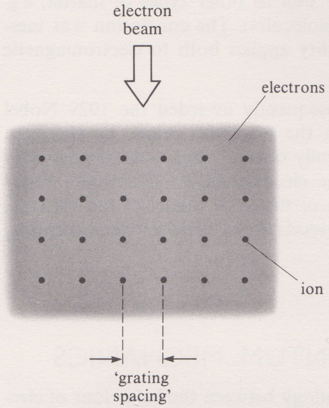

FIGURE 18 A metallic crystal may be regarded as a diffraction grating—the spacing of the ions in the crystal is, loosely speaking, the 'grating spacing'. This illustration shows only a 'slice' through the crystal, which is of course actually three-dimensional.

The question of whether electrons can be diffracted by metallic crystals was investigated in 1925 by Clinton Davisson and his junior collaborator Lester Germer at the Bell Telephone Laboratories in New York. In 1928, G. P. Thomson (the son of J. J. Thomson) independently investigated the question at the University of Aberdeen. The apparatus used by Thomson was similar to the one shown schematically in Figure 19: electrons are accelerated through several hundred volts, propagate through a target of metal foil and are then detected.

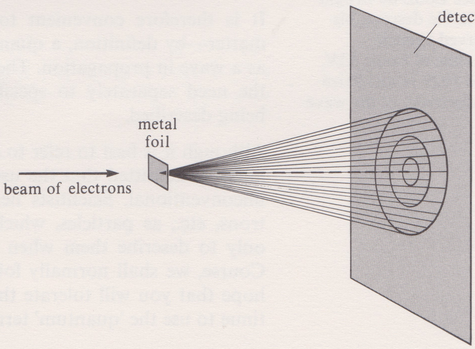

FIGURE 19 Schematic diagram of the type of apparatus used by G. P. Thomson when he observed the diffraction of electrons.

WAVE–PARTICLE DUALITY

QUANTUM

QUANTUM MECHANICS

It was found that the electrons were indeed diffracted and that the observed diffraction patterns were perfectly consistent with expectations based on the de Broglie formula $\lambda_{\mathrm{dB}} = h/p$ (e.g. Figure 20a). Moreover, these patterns resemble closely the ones obtained using similar apparatus that used, instead of electrons, a beam of X-rays, i.e. electromagnetic radiation (Figure 20b). This illustrates clearly that diffraction of matter and diffraction of radiation are precisely analogous phenomena.

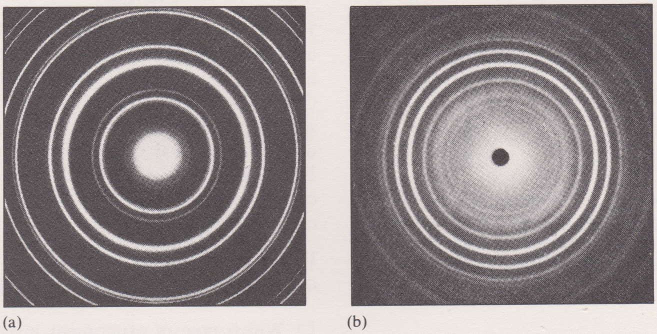
(a) (b)

FIGURE 20 (a) A diffraction pattern obtained using a beam of electrons and a target of gold crystals, in the configuration shown in Figure 19. (b) A diffraction pattern obtained using a beam of X-rays and a target of zirconium oxide crystals, also in the configuration shown in Figure 19.

Thus, de Broglie's revolutionary idea that a wave is associated with each sample of free matter was vindicated, at least for electrons (it was later checked that the idea applied equally well to other types of matter, e.g. protons, neutrons, nuclei, atoms and molecules). The conclusion was inescapable: the idea of **wave–particle duality** applies both to electromagnetic radiation *and* matter.

Louis de Broglie (Figure 21) was subsequently awarded the 1929 Nobel Prize for physics and, eight years later, the Prize was shared by Davisson and G. P. Thomson. The Thomson family certainly did make diverse contributions to our understanding of the electron. G. P. Thomson's Nobel Prize was awarded for his verification of the *wave* model of the electron, whereas his father, J. J. Thomson, had received his Prize for demonstrating that the electron behaves as a *particle*!

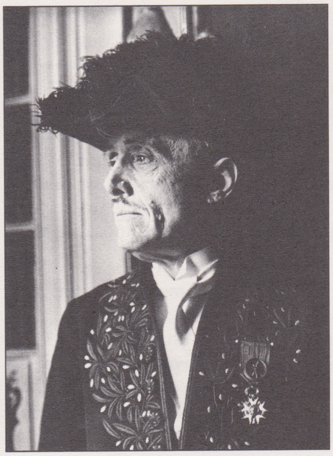

FIGURE 21 Prince Louis de Broglie (1892–1987) was of noble descent, his ancestors having served French monarchs since the time of Louis XIV. He was awarded the 1929 Nobel Prize for physics for 'his discovery of the wave nature of electrons'.

2.3 QUANTA AND QUANTUM MECHANICS

You have now seen that there is an analogy between the behaviour of electromagnetic radiation and that of matter. A particle model is required to describe their interactions, whereas a wave model is needed to describe their propagation. Hence, there is a problem of terminology: it is not strictly correct to describe matter (electrons, protons, neutrons, etc.) and electromagnetic radiation simply as 'particles' or as 'waves'. These terms each apply only in certain circumstances ('particles' for their interactions, 'waves' for their propagation).

It is therefore convenient to apply the term **quantum** to radiation *and* matter—by definition, a quantum behaves as a particle in interactions and as a wave in propagation. The advantage of using this term is that it avoids the need separately to specify whether propagation or an interaction is being described.

Although it is best to refer to electrons, protons, neutrons, etc., as quanta, it must be admitted that the use of the term to describe matter is somewhat unconventional. Scientists nearly always refer to electrons, protons, neutrons, etc., as particles, which of course should strictly speaking be used only to describe them when they interact. In the remaining Units of the Course, we shall normally follow this widely accepted convention, and we hope that you will tolerate this laxity. However, in this Unit we shall continue to use the 'quantum' terminology.

You may already have wondered whether there exists a theory that can describe both the propagation *and* interactions (i.e. wave and particle behaviour) of matter, just as the theory of quantum electrodynamics can describe the propagation and interaction of radiation (Unit 10). Such a theory has been formulated—it is called **quantum mechanics**. Sometimes the term *quantum theory* is used as a synonym for quantum mechanics, but we shall use the former term in a more general sense to encompass all the theoretical concepts that apply to all quanta (i.e. matter *and* radiation).

In the remainder of this Unit, some of the basic concepts of quantum mechanics will be described and then, in Unit 31, they will be applied to the atom and the nucleus.

SUMMARY OF SECTION 2

1 The propagation of electromagnetic radiation is described by a wave model, which enables the diffraction of radiation to be understood.

2 The interactions of electromagnetic radiation with matter can be described by a particle model in which each quantum of radiation (photon) has an energy $E = hf$ and a momentum of magnitude $p = E/c$, in the usual notation. This model enables the photoelectric effect and the Compton effect to be understood.

3 The propagation of free matter is described by de Broglie's wave model, according to which free matter has an associated wavelength $\lambda_{\mathrm{dB}} = h/p$, in the usual notation. This model successfully predicted that electrons can be diffracted.

4 The interactions of matter can be understood using a particle model.

5 The idea of wave–particle duality applies to both electromagnetic radiation and matter.

6 Quantum mechanics is a theory that describes the wave and particle behaviour of matter.

SAQ 2 (a) A student doing the physics experiment at Summer School arranges for a collimated beam of sodium light with a wavelength of 589 nm to impinge perpendicularly on a diffraction grating whose spacing is 1 670 nm. At what angle will the $n = 1$ diffraction maximum be observed?

(b) At what angle would the $n = 1$ diffraction maximum have occurred if the student had used, instead of light, a collimated beam of free electrons with a speed of $1.00 \times 10^{6}\,\mathrm{m\,s^{-1}}$? (Take the mass of the electron as $9.11 \times 10^{-31}\,\mathrm{kg}$, and remember that $1\,\mathrm{J} = 1\,\mathrm{kg\,m^{2}\,s^{-2}}$.)

SAQ 3 A beam of X-rays has a frequency of 10^{18} Hz. (Remember that $1\,\mathrm{Hz} = 1\,\mathrm{s^{-1}}$).

(a) What is the energy of each X-ray photon?

(b) What is the momentum of each X-ray photon?

SAQ 4 Fill in the two missing bits of evidence in Table 2 (*overleaf*).

SAQ 5 Which *two* of the following statements about quantum mechanics are correct?

(a) It can describe the propagation of visible light.

(b) It can describe the interactions of electrons.

(c) It can describe the propagation of electrons.

(d) It applies only to electrons.

TABLE 2 Summary of experimental evidence for the particle and wave models of electrons and electromagnetic radiation (SAQ 4)

Model	Experimental evidence
particle model of the electron	☐ the electron has a definite charge and mass ☐ an electron can leave a clearly defined track in a bubble chamber
wave model of the electron	☐
particle model of electromagnetic radiation	☐ photoelectric effect ☐
wave model of electromagnetic radiation	☐ diffraction (e.g. the physics experiment at Summer School)

3 QUANTUM MECHANICAL DESCRIPTIONS OF MATTER

In this Section, the extraordinary nature of the type of wave associated with matter will be described. Then, in the AV sequence 'Wavefunctions of matter', the waves associated with matter in different circumstances (free and confined) will be illustrated and discussed.

3.1 WHAT TYPE OF WAVE IS ASSOCIATED WITH MATTER?

Consider the experiment shown schematically in Figure 22: a beam of free electrons (i.e. matter) impinges on a single slit and a diffraction pattern is observed on a screen, which serves as a detector. This is precisely analogous to the observation that light is diffracted when it is shone on a single slit (Unit 10).